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Forward elastic scattering of light on light, $\gamma + \gamma \to \gamma + \gamma$

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Abstract. The forward elastic scattering of light on light, *i.e.,* the reaction $\gamma+\gamma\to\gamma+\gamma$ in the forward direction, is analyzed utilizing real analytic amplitudes. We calculate $\rho_{\gamma\gamma}$, the ratio of the real to the imaginary portion of the forward scattering amplitude, by fitting the total $\gamma\gamma$ cross section data in the high energy region 5 GeV $\le \sqrt{s} \le$ 130 GeV, assuming a cross section that rises asymptotically as $\ln^2 s$. We then compare $\rho_{\gamma\gamma}$ to ρ_{nn}, the ratio of the *even* portions of the pp and $\bar{p}p$ forward scattering amplitudes, as well as to $\rho_{\gamma p}$ [1], the ρ value for Compton scattering. Within errors, we find that the three ρ-values in the c.m.s. energy region 5 GeV $\le \sqrt{s} \le$ 130 GeV are the same, as predicted by a factorization theorem of Block and Kaidalov [2].

The purpose of this note is to analyze light on light elastic scattering at high energies, *i.e.,* $\gamma+\gamma\to\gamma+\gamma$, when the scattered γ is in the forward direction, in order to extract $\rho_{\gamma\gamma}$, the ratio of the real to the imaginary portion of the forward scattering amplitude, by using real analytic amplitudes. To the best of our knowledge, no one has either measured or calculated $\rho_{\gamma\gamma}$.

Block and Kaidalov [2] have shown that $\rho_{\rm nn} = \rho_{\gamma p} = \rho_{\gamma\gamma}$ if one uses eikonals for $\gamma\gamma$, γp and the even portion of nucleon-nucleon scattering that have *equal* opacities, *i.e.,* eikonals that have the same value at impact parameter $b = 0$ (for details of the assumptions made, see ref. [2]). This is the equivalent of the more physical statement that

$$\left(\frac{\sigma_{\rm elastic}(s)}{\sigma_{\rm tot}(s)}\right)_{\gamma\gamma} = \left(\frac{\sigma_{\rm elastic}(s)}{\sigma_{\rm tot}(s)}\right)_{\gamma p} = \left(\frac{\sigma_{\rm elastic}(s)}{\sigma_{\rm tot}(s)}\right)_{nn}, \quad \text{for all } s. \tag{1}$$

Block and Kaidalov [2] have proved three factorization theorems:

$$\frac{\sigma_{\rm nn}(s)}{\sigma_{\gamma \rm p}(s)} = \frac{\sigma_{\gamma \rm p}(s)}{\sigma_{\gamma\gamma}(s)}, \tag{2}$$

where the σ's are the total cross sections for nucleon-nucleon, γp and $\gamma\gamma$ scattering,

$$\frac{B_{\rm nn}(s)}{B_{\gamma \rm p}(s)} = \frac{B_{\gamma \rm p}(s)}{B_{\gamma\gamma}(s)}, \tag{3}$$

where the B's are the nuclear slope parameters for elastic scattering, and

$$\rho_{\rm nn}(s) = \rho_{\gamma \rm p}(s) = \rho_{\gamma\gamma}(s), \tag{4}$$

where the ρ's are the ratio of the real to imaginary portions of the forward scattering amplitudes, with the first two factorization theorems each having their own proportionality constant. These theorems are exact, for *all* s (where $\sqrt{s}$ is the c.m.s. energy), and survive exponentiation of the eikonal (see ref. [2]). We emphasize that the equality of the three ρ values, the theorem of (4), is valid *independently* of the model which takes one from nn to γp to $\gamma\gamma$ reactions, as long as the respective eikonals have equal opacities, *i.e.,* (1) holds. Block [1] has already shown that $\rho_{\rm nn}(s) = \rho_{\gamma p}(s)$.

The purpose of this communication is to demonstrate using available high energy experimental data, that within errors, $\rho_{\rm nn}(s) = \rho_{\gamma\gamma}(s)$, completing the validation of the last factorization theorem given in (4). However, no direct measurements are available for $\rho_{\gamma\gamma}(s)$. We will find $\rho_{\gamma\gamma}(s)$ utilizing an analysis involving real analytic amplitudes, a technique first proposed by Bourrely and Fischer [3]. This work follows the procedures and conventions used by Block and Cahn [4]. The variable s is the square of the c.m. system energy, whereas ν is the laboratory system momentum. In terms of the *even* laboratory scattering amplitude f_+, where $f_+(\nu) = f_+(-\nu)$, the total unpolarized $\gamma\gamma$ cross section $\sigma_{\gamma\gamma}$ is given by

$$\sigma_{\gamma\gamma} = \frac{4\pi}{\nu}{\rm Im} f_+(\theta = 0), \tag{5}$$

where θ is the laboratory scattering angle. We further assume that our amplitudes are real analytic functions with a simple cut structure [4]. We use an even amplitude for $\gamma\gamma$ reactions in the high energy region, far above any cuts, (see ref. [4], p. 587, (5.5a), with $a = 0$), where the even amplitude simplifies considerably and is given by

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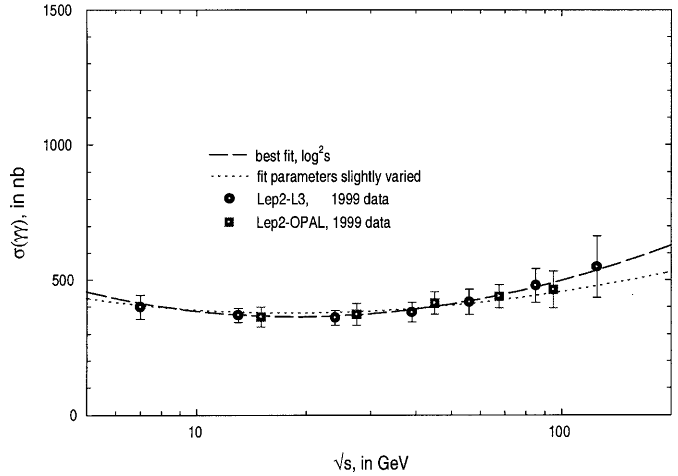

Fig. 1. The dashed curve is $\sigma_{\rm tot}(\gamma\gamma)$, the predicted total $\gamma\gamma$ cross section (in nb) from (9), using the central value parameters of a χ^2 fit, $A = 377$ nb, $\beta = 11.1$ nb, $s_0 = 266.7$ GeV^2, with $\mu = 0.5$ and $c = 220$ nbGeV, compared to the existing high energy experimental data in the c.m.s. energy interval 5 GeV $\le \sqrt{s} \le 130$ GeV. The dotted curve varies the parameters slightly, with $s_0 \to 189$ GeV^2 and $\beta \to 5.9$ nb (values within their errors). The corresponding $\rho_{\gamma\gamma}$ curves are shown in Fig. 2

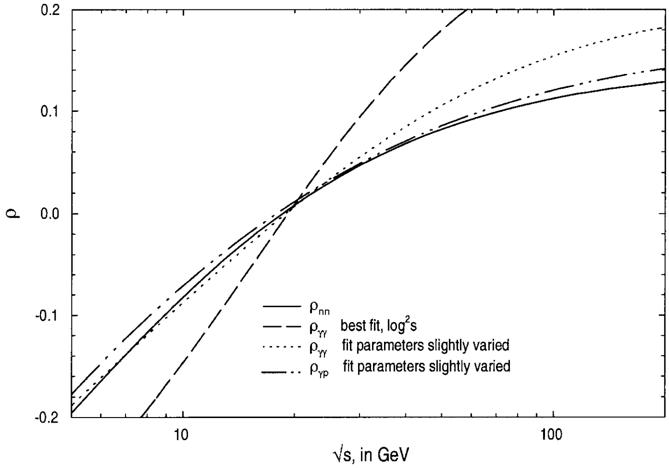

Fig. 2. The solid curve is $\rho_{\rm nn}$, the predicted ratio of the real to imaginary part of the forward scattering amplitude for the 'elastic' reactions, $\gamma+\gamma \to V_i + V_j$ scattering amplitude, where $V_{i,j}$ is ρ, ω or ϕ (using factorization). The dashed curve is $\rho_{\gamma\gamma}$, the ratio of the real to imaginary part of the forward scattering amplitude for elastic scattering, $\gamma+\gamma\to\gamma+\gamma$, found from (10) using real analytic amplitudes that asymptotically go as $\ln^2 s$, with best fit parameters $A = 377$ nb, $\beta = 11.1$ nb, $s_0 = 266.7$ GeV^2, with $c = 220$ nbGeV and $\mu = 0.5$. The dotted line is the $\rho_{\gamma\gamma}$ curve where the parameters of the fit have been slightly varied within their errors, with $s_0 \to 189$ GeV^2 and $\beta \to 5.9$ nb. The corresponding two curves for $\sigma_{\rm tot}(\gamma\gamma)$ are shown in Fig. 1. The dashed dot dot curve is $\rho_{\gamma p}$, taken from ref. [1] and is shown for comparison with $\rho_{\gamma\gamma}$ and $\rho_{\rm nn}$

$$f_+(s) = i\frac{\nu}{4\pi}\Big\{A + \beta[\ln(s/s_0) - i\pi/2]^2 + cs^{\mu-1}e^{i\pi(1-\mu)/2}\Big\}, \tag{6}$$

where A, β, c, s_0 and μ are real constants. We are ignoring any real subtraction constants. In (6), we have assumed that the $\gamma\gamma$ cross section rises asymptotically as $\ln^2 s$. The real and imaginary parts of (6) are given by

$$\mathrm{Re}\frac{4\pi}{\nu}f_+(s) = \beta\,\pi\ln s/s_0 - c\,\cos(\pi\mu/2)s^{\mu-1} \tag{7}$$

$$\mathrm{Im}\frac{4\pi}{\nu}f_+(s) = A + \beta\left[\ln^2 s/s_0 - \frac{\pi^2}{4}\right] + c\,\sin(\pi\mu/2)s^{\mu-1}. \tag{8}$$

Using equations (5), (7) and (8), the total cross section for high energy $\gamma\gamma$ scattering is given by

$$\sigma_{\rm tot}(s) = A + \beta\left[\ln^2 s/s_0 - \frac{\pi^2}{4}\right] + c\,\sin(\pi\mu/2)s^{\mu-1}, \tag{9}$$

and ρ, the ratio of the real to the imaginary portion of the forward scattering amplitude, is given by

$$\rho(s) = \frac{\beta\,\pi\ln s/s_0 - c\,\cos(\pi\mu/2)s^{\mu-1}}{\sigma_{\rm tot}}. \tag{10}$$

If we assume that the term in c is a Regge descending term, then $\mu = 1/2$.

Total cross sections for $\gamma\gamma$ scattering are now available for c.m.s. energies up to $\approx$130 GeV [5]. We have made a χ^2 fit of (9) to the experimental $\sigma_{\rm tot}(\gamma\gamma)$ data in the c.m.s. energy interval 5 GeV $\le \sqrt{s} \le 130$ GeV. We find a reasonable representation of the data using (9), with a χ^2 per degree of freedom of 0.066 for 9 degrees of freedom, with the coefficients:

$$A = 377 \pm 16\,\mathrm{nb},\ \beta = 11.1 \pm 5.3\,\mathrm{nb},$$
$$s_0 = 266.7 \pm 157\,\mathrm{GeV}^2,$$

using the fixed values, $c = 220$ nbGeV and $\mu = 0.5$. The cross sections of (9) are in nb, with s in GeV^2. This fit, plotted as a function of c.m.s. energy, gives the dashed cross section curve $\sigma_{\rm tot}(\gamma\gamma)$ in Fig. 1, as well as the dashed $\rho_{\gamma\gamma}$ curve in Fig. 2, using (9) and (10), respectively. Several comments about both the experimental data and the fit are in order. First, the two independent cross section measurements of L3 and OPAL have been utilized, which are separately normalized using Monte Carlos. Second, the expected χ^2/d.f. is 1, whereas the value obtained in our fit is only 0.066 – a very unlikely result if the experimental points are uncorrelated and have their statistical errors allocated correctly. Clearly, more precise data is required to settle this issue, as well as reducing the (rather large) errors of the fitted parameters. Third, it should be emphasized that the ρ value (see (10)) is independent of the absolute normalization of the total cross section $\sigma_{\rm tot}$, depending only on its shape. Clearly, relative normalization errors between the OPAL and L3 experiments play a critical role in shape dependence, whereas overall normalization does not affect the result for ρ. Fourth, if the cross sections, e.g., for $\sigma_{\rm tot}$(nn) and $\sigma_{\rm tot}(\gamma\gamma)$ are proportional to each other for large s, i.e., have the same *shape*, then inspection of (6), (9) and (10) shows that $\rho_{\rm nn}(s) = \rho_{\gamma\gamma}(s)$. In other words, if the cross sections are similar, within experimental errors, then the ρ-values will be equal, within errors.

In order to demonstrate the sensitivity of $\sigma_{\rm tot}(\gamma\gamma)$ and $\rho_{\gamma\gamma}$ to the parameters of the fit, we have also plotted in Fig. 1 the dotted curve (a variation of the parameters of s_0 and β within their errors), where we have set $s_0 = 189$ GeV^2 and $\beta = 5.9$. This curve has as its $\rho_{\gamma\gamma}$ analog the dotted curve of Fig. 2. Using (10), Fig. 2 shows our result for $\rho_{\gamma\gamma}$ compared to ρ_{nn}, the ρ-value for the even portion of nucleon-nucleon scattering found in ref. [6], as a function of the c.m.s. energy $\sqrt{s}$, in GeV. The solid curve is ρ_{nn}; the dashed line is the $\rho_{\gamma\gamma}$ curve which corresponds to the central values $A = 377$ nb, $\beta = 11.1$ nb, $s_0 = 266.7$ GeV^2, with $c = 220$ nbGeV and $\mu = 0.5$; the dotted line is the $\rho_{\gamma\gamma}$ curve which uses the slightly varied parameters $s_0 = 189$ GeV^2 and $\beta = 5.9$ nb. Also shown as the dashed dot dot curve in Fig. 2 is $\rho_{\gamma p}(s)$, taken from a recent analysis of Compton scattering [1]. The agreement between the slightly modified $\rho_{\gamma\gamma}(s)$ and $\rho_{nn}(s)$ over the energy interval 5 GeV $\leq \sqrt{s} \leq$ 130 GeV, as well as with as with $\rho_{\gamma p}(s)$, lends experimental support – in a model independent way – for the three factorization theorems of Block and Kaidalov [2,7], $\sigma_{\rm nn}(s)/\sigma_{\gamma {\rm p}}(s) = \sigma_{\gamma {\rm p}}(s)/\sigma_{\gamma\gamma}(s)$, $B_{\rm nn}(s)/B_{\gamma {\rm p}}(s) = B_{\gamma {\rm p}}(s)/B_{\gamma\gamma}(s)$, and $\rho_{\rm nn}(s) = \rho_{\gamma {\rm p}}(s) = \rho_{\gamma\gamma}(s)$. Clearly, these conclusions would be greatly strengthened by precision cross section measurements of both γp and $\gamma\gamma$ reactions at high energies.

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